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Particles, Waves, and the Interpretation of Quantum Mechanics

What are the electrons really like? Are they like particles or waves? Like both particles *and* waves, or like neither? This question, so frequently asked in an introductory course of quantum theory, illustrates the psychological difficulties with which students are confronted in trying to learn the concepts of quantum mechanics. For while the objects of microphysics cannot be completely comprehended by means of analogies taken from ordinary experience (i.e., classical models) the very notion of explanation implies the use of such analogies.

What then would be an intuitively satisfactory approach to the theory? Ordinarily the subject is taught as a system of experimental and mathematical principles without many references to its conceptual structure. The students are initially supposed to accept the rules at face value because they work and, as is usually the case, by the time they learn how to practice them, they forget their problematic character.

In what follows an attempt is made for a brief description of the conceptual basis of quantum mechanics in the light especially of the commonly accepted "orthodox" interpretation, developed by the scientists of the Copenhagen School such as Bohr, Heisenberg, Bonn, Pauli, Jordan, etc. It is hoped that the discussion, despite its limited scope, will provide an insight into the logical structure of quantum mechanics and its philosophical implications.

Quantum Mechanics

What is quantum mechanics? Briefly, it is the study of the behavior of matter at the atomic and subatomic levels, which involve particles of very small rest mass and very small volume—electrons, protons, neutrons, etc. Photons, and in fact all particles of zero mass, because they always travel with the velocity of light are, of course, relativistic entities and as such are subject to the laws of relativistic quantum mechanics. However, as far as the considerations of this discussion go, they may as well be included in the category of electrons.

The formalism of quantum mechanics is based on the consideration that every state of a quantum-mechanical system can be described at a given instant by a specific coordinate function—the wavefunction ψ—which is a physically appropriate solution of Schrödinger's wave equation. For a compound system consisting of many particles the wavefunction is given in terms of the coordinates of all particles; therefore, it is a function of a point, not in real physical space, but in a multidimensional configuration space. The wavefunction is usually assumed to represent the amplitude of the wave field (deBroglie wave) associated with the quantum-mechanical system, and as such it provides the total possible information about the state of the system. How?

In the first place, the square of its absolute value (which is the intensity of the wave field) is taken to represent the probability distribution of the coordinates of the system, and for the case of a single particle the probability per unit volume of finding it in a given place and at a certain time-instant. The probabilistic interpretation is forced upon us by the necessity to reconcile the wave and particle properties of matter. Secondly, the wavefunction, by dint of the superposition principal (which, among other things, is reflected in the linearity property of Schrödinger's equation) can in general be split up into a number of partial solutions, each representing a stationary wave of characteristic frequency or, for that matter, a stationary state of definite energy. The probability and superposition principles embody essentially the spatial distribution and quantization of energy of a microscopic system. For, when the state of a system is represented by a certain wavefunction, the energies corresponding to the functions into which the original function is decomposable represent the possible quantized energy values of the system, whereas the probabilities associated with these functions denote the chance that each of these energies can be obtained.[1]

Furthermore, the possibility of identifying the velocity of a particle with the group velocity of its deBroglie wave (*1*) allows us to utilize the well-known Fourier's relation, $\Delta x \, \Delta(1/\lambda) \geq 1$, (where Δx is the extension of the wave packet in space and $\Delta(1/\lambda)$ is the wave number interval of the packet) in conjunction with the deBroglie relation, $p = h/\lambda$, and obtain $\Delta p \cdot \Delta x \geq h$. This, of course, is the famous Heisenberg's uncertainty relation which, moreover, applies for any pair of conjugate variables such as time and energy, etc. What does it mean? It simply means that the product of the indefiniteness of the particle's position (defined by the length of the wave packet within which the particle is assumed to exist) and the indefiniteness of its momentum can never exceed h. The immediate practical conclusion of the above is that we cannot measure simultaneously the values of two conjugate variables with accuracies exceeding the limits prescribed by Heisenberg's relation. For example, the more accurately the position of a particle is measured, the less accurate the value of its corresponding momentum p_x becomes so that the product of their uncertainties is greater or at least equal to h. This limitation is not to be ascribed to any imperfection of the experimental techniques, but is an inherent property of nature associated with the very existence of the quantum of action.[2]

The Copenhagen Interpretation

It is easy to see from the preceding statements that the wave-particle duality, the probability character of the wavefunction, and the uncertainty relations are three different aspects of the same wave field associated with the microparticle, and as such they could, conceivably, be better comprehended by a more profound analysis of its physical structure. Now we raise the question: What is the physical structure of this field? From the outset it is almost apparent that the deBroglie wave does not possess the degree of physical reality which the electromagnetic wave, for example, has. For though it develops in space and time continuously and in a causal manner as do classical fields, its amplitude—the wavefunction—which is usually a complex quantity in a multidimensional space, is as much subjective as it is objective. For one thing, the wavefunction can be modified (normalized) according to our knowledge about the possible whereabouts of the particle and changes

[1] Similar conclusions can be drawn about all physical observables (e.g., position, momentum, etc.) by virtue of their associability with appropriate operators.

[2] For illustrations of the uncertainty principle see Ref. (*2*).

abruptly as a result of observation (reduction of the wave packet). For another, by virtue of its association with the notion of probability which expresses partial ignorance, it may be considered only partially objective. Paraphrasing Heisenberg slightly, we may say that the wavefunction does not represent a course of events in space and time but rather a tendency for events and our knowledge of events. It may therefore be considered objective only to the extent this tendency is realized by some observation or measurement (*3*). These features are not common to the classical field.

Thus, it appears that the quantum mechanical wave field is subjective, though it does possess objective features or, at any rate, the tendency for exhibiting such features. But how, or under what conditions, are these features manifested? They are manifested during the act of observation, when, for instance, a particle localization on a photographic plate takes place, and then only partially and one-sidedly. At the same time the wavefunction suddenly becomes zero in every part of space beyond the place in which the particle is observed. The act of observation, in other words, seems to effect two things: on the one hand, it forces the wave field to reveal part of its nature (potential or otherwise), and on the other hand, it suddenly disrupts its causal development. Let us try to be a little more explicit on these.

It is a characteristic of classical theory to utilize for the description of the state of a particle magnitudes, such as the coordinates of position and of momentum, which cannot in quantum mechanics be accurately defined at the same time. For example, to define exactly the state of a particle at a given time we need to know exactly its position and momentum. Now, however, if the position is fixed, i.e., if the wave packet associated with the particle is shrunk to a point, the momentum can have all possible values, according to the well-known Fourier theorem. Conversely, a fixed momentum, which implies an infinitely long monochromatic wave, means that the particle can exist in a region of infinite extension. Consequently, the indefiniteness of the position of a particle cannot change independently of the indefiniteness of its momentum.

One might suggest that the practical division of state parameters can be seen if we examine the roles of the measuring instruments (*4*). An instrument which is suitable for measuring positions, for instance, must consist of rigid scales, whereas the instrument required for measuring impulses must contain movable parts. These two features, apparently, cannot coexist in a given experimental setup at one and the same time. It follows therefore that position and momentum cannot be simultaneously measured with absolute accuracy.

The above limitations, which of course apply in classical physics as well, bring forth the following question: Would it not be possible to accurately measure position and momentum in the manner that is theoretically feasible in classical physics, that is to say, by taking into consideration all appropriate experimental and theoretical corrections? This question, however, should be answered in the negative if we realize that any experimental observation involves not only an action from the object to the observer but also a counteraction in the opposite way. This counteraction becomes especially noticeable by the minute and sensitive particles of the microworld and, what is more, it cannot be predicted or compensated for or, at any rate, minimized beyond a certain limit because of the existence of the quantum of action. The notion of "limiting perturbation" is very important in quantum theory because, on the one hand, it provides a basis of distinction between quantum-mechanical objects and classical objects (which can in principle be observed without being perturbed), and on the other hand, it emphasizes the requirement that any rational explanation of quantum mechanics must be based on an adequate theory of measurement. So much for that.

At this point, it is pertinent to come back to the uncertainty principle and briefly explain some of the consequences that follow directly from it. In the first place, because it makes no sense to talk about the exact position of a particle with definite velocity the concept of a particle's trajectory (or orbit) is basically inadmissible in quantum mechanics. This means, moreover, that the particle has, in itself, no other definite classical characteristic. But quantum mechanics cannot be formulated without classical concepts. It is quite meaningless without them. For any quantitative description of the motion of a quantum-mechanical object presupposes *measurement*—namely, the process of interaction with a classical object, the changes of which (described accurately by means of classical concepts) are used to determine the conditions of the former. This points to the special place which quantum mechanics occupies among physical theories. Usually, a general theory can be formulated in a logically consistent manner from its fundamental premises with no reference to another "not-so-general" theory representing its limiting case. For example, relativistic mechanics can be formulated independently of the principles of Newtonian mechanics. But quantum mechanics, though including classical mechanics as a limiting case, depends on it for its own substantiation.

Now, one may raise the question of whether the uncertainty relations have merely epistemological or ontological significance, that is to say, whether the particle at any given moment occupies a definite place and has a definite velocity which nevertheless cannot be exactly ascertained, or it occurs simultaneously along the whole extent of space in which its wavefunction is not zero and has all probable values of velocity. Despite the fact that the overwhelming majority of physicists instinctively adhere to the first view, it is essential to recognize that in the Copenhagen interpretation the second alternative is preferred or, at any rate, the question itself is brushed aside as a physically inappropriate one. In Heisenberg's view, for example, it is meaningless to speak or speculate about what happens between observations since "we have to realize that the word 'happens' can apply only to observation, not the state of affairs between two observations" (*5*). In fact, it is not necessary to speak of particles at all since in many instances the concept of matter waves is more suitable to account for the experimental observations. The notions of particle or wave have objective meaning only to the extent they are related to an eventual measurement; in all other situations, they merely signify the sum total of the potentialities of measurement contained in the wavefunction. It is needless to say that this positivistic understanding of the matter constitutes a significant departure from the classical ideal of objective reality.

The second consequence which, incidentally, lies at the roots of quantum statistics and the modern theory of chemical bonding deals with the notions of individuality (or sameness) and distinguishability of atomic objects.[3] In classical atomic theory the atoms are assumed to be numerable, small bodies, possessing definite identity like the ordinary, palpable objects. Even in the absence of intrinsic, qualitative differences, they can in principle be characterized and identified in terms of their specific locations in the space-time continuum; their motion can be represented by definite non-overlapping trajectories, uniquely determined from the equations of motion.

Now, in quantum mechanics, the breakdown of the concept of trajectory and the recognition of the fact that it is impossible to obtain a continuous, gapless description of atomic phenomena leads to the abandonment of the idea of distinguishability of 'like' particles. This implies that between two similar particles found in a small region of space

[3] For a detailed discussion of this idea see, for example, Ref. (*6*).

we have no way of telling 'which is which,' and, what is more, that two consecutive observations of a micro-object (even if we have reason to believe that they are causally connected) cannot justify the claim that they refer to exactly the same thing. In short, we are compelled to dismiss the idea of individuality of microparticles as basically meaningless and to assume that the particular observations of atomic objects should be regarded as isolated events. This leads to some intriguing questions: If the fundamental constituents of matter have no individuality, how are we to explain the apparent individuality of macroscopic objects? In terms of structure and shape, primarily? If so, is form a more fundamental concept than substance? We will not pursue these questions further, but instead we will refer the interested reader to some appropriate places (*7*).

We are now in a position to see the reason why from the standpoint of the Copenhagen interpretation the principle of determinism does not apply in the microworld.[4] For if the states of a system cannot be determined exactly, the establishment of definite causal links between events is impossible, and consequently determinism becomes operationally unverifiable. Therefore, one might as well assume that it does not exist. To the question whether the principle might still be applicable in atomic mechanics in terms of a formulation that would involve nonobservable parameters, as we shall see shortly in discussing the complementarity principle, the following answer is given: Either we describe atomic phenomena in space and time by means of the terminology of classical physics—and in that case determinism does not hold because of the uncertainty relations—or we describe them solely in terms of the wave function ψ which develops in space and time continuously and in accordance with causal laws. But one must remember, however, that ψ is only partially definable from the square of its modulus $|\psi|^2$ which has real significance. Consequently, even if $|\psi|^2$ were known at some initial time, ψ could not be determined exactly. This amounts to the assertion that events follow a deterministic line, but we do not know where it begins—*viz.*, the initial states. In other words, even if a hidden determinism exists, it is bound to stay hidden forever.

The reader by now may perhaps be ready to raise the following objection: In view of the fact that the classical concepts are found to be inadequate in explaining atomic phenomena, it is unwarranted to decide on the basis of these concepts alone whether the law of determinism stands or falls; it would be more reasonable to admit that the law cannot be accurately transcribed by means of the classical concepts rather than to deny its validity altogether. In the Copenhagen interpretation, however, this objection is dismissed on the grounds that it overlooks the reasons which make classical notions necessary. The concepts of classical physics are just a refinement of the concepts of our sense-perceptions by which we come to "see" directly how nature behaves. Therefore, to describe our experimental arrangements and communicate unambiguously the results of our observations we must use classical language. Granted, the classical concepts do not fit nature accurately, but it is utopian to think that we can (or should) replace them.

It was Bohr (*8*) who first realized that the aforementioned dilemma in which quantum theory found itself could not be resolved by reinterpreting traditional notions or introducing new ones. He concluded that what was needed was a new idea that could lead to an understanding of the underlying logic of the theory. Bohr called this idea "complementarity" (later renamed complementarity principle) to denote the logical relation between two different modes of description which, though mutually incompatible, are both necessary for an exhaustive description of the situation. These modes never lead to contradictions because every experimental circumstance that requires the application of the one denies the possibility of applying the other. For example, the wave and corpuscular pictures were said by Bohr to be complementary because both are necessary for a complete description of microparticles, and each is possible at the exclusion of the other. Moreover, by the appropriate application of these pictures, which appear under different experimental conditions, one can get the right impression of what lies behind the atomic experiments. By a similar reasoning, Bohr arrived at a complementarity relationship between the spatiotemporal and causal descriptions of phenomena.

Implicit in the notion of complementarity is the indeterminateness of the concept of "observation." In fact, it is the reason which permits (and even makes necessary) the use of complementary concepts in the description of atomic phenomena. Because it is impossible to eliminate completely the interaction between the atomic objects and the means of observation, "evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the subjects" (*9*).

It is important to realize that the complementarity principle, despite the ambiguity of its definition, provides often an effective means for explaining the peculiarities of quantum-mechanical formalism, and for that matter is considered today the basic methodological principle of the Copenhagen interpretation. But it is also fair to say that this idea and, indeed, the whole explanation of quantum mechanics provided by the Copenhagen School have met with serious objections from many philosophically minded physicists such as Planck, Einstein, von Laue, deBroglie, Schrödinger—to name a few. An examination of these objections, however, even in a very limited way, would require the space of another paper.

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[4] The principle of determinism might be expressed by the statement that given sufficient data regarding the state of the system (e.g., the coordinates of position and momentum) one can in principle predict (or retrodict) unambiguously the state of the system at any other time by the appropriate application of the laws of motion. This of course is not the most general definition of the principle, but, in our opinion, it is the one implied in the Copenhagen interpretation.